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Assignment 1.1 Fractions and Decimals Worksheet:

Name: Claire Date: 9.4

1. Convert each of the following fractions to a percentage without using a calculator.

Denominator 2

Fraction	Decimal
$\frac{1}{2}$	0.5
$\frac{2}{2}$	1.
$\frac{3}{2}$	1.5
$\frac{4}{2}$	2
$\frac{5}{2}$	2.5
$\frac{6}{2}$	3.

Denominator 3

Fraction	Decimal
$\frac{1}{3}$	$0.\overline{333}$
$\frac{2}{3}$	$0.\overline{666}$
$\frac{3}{3}$	1
$\frac{4}{3}$	$1.\overline{333}$
$\frac{5}{3}$	$1.\overline{666}$.
$\frac{6}{3}$	2.

Denominator 4

Fraction	Decimal
$\frac{1}{4}$	0.25
$\frac{2}{4}$	0.5
$\frac{3}{4}$	0.75
$\frac{4}{4}$	1
$\frac{5}{4}$	1.25
$\frac{6}{4}$	1.5

Denominator 5

Fraction	Decimal
$\frac{1}{5}$	0.2.
$\frac{2}{5}$	0.4
$\frac{3}{5}$	0.6
$\frac{4}{5}$	0.8
$\frac{5}{5}$	1.
$\frac{6}{5}$	1.2.
$\frac{7}{5}$	1.4
$\frac{8}{5}$	1.6

Denominator 6

Fraction	Decimal
$\frac{1}{6}$	$0.1\overline{666}$
$\frac{2}{6}$	$0.\overline{333}$
$\frac{3}{6}$	0.5.
$\frac{4}{6}$	$0.\overline{666}$
$\frac{5}{6}$	$0.8\overline{333}$
$\frac{6}{6}$	1
$\frac{7}{6}$	$1.1\overline{666}$
$\frac{8}{6}$	$1.\overline{333}$

Denominator 7

Fraction	Decimal
$\frac{1}{7}$	$0.\overline{142857}$
$\frac{2}{7}$	$0.\overline{285714}$
$\frac{3}{7}$	$0.\overline{428571}$
$\frac{4}{7}$	$0.\overline{571428}$
$\frac{5}{7}$	$0.\overline{714285}$
$\frac{6}{7}$	$0.\overline{857142}$.
$\frac{7}{7}$	1
$\frac{8}{7}$	$1.\overline{142857}$.

Denominator 8:

Fraction	Decimal
$\frac{1}{8}$	0.125
$\frac{2}{8}$	0.25
$\frac{3}{8}$	0.375
$\frac{4}{8}$	0.5
$\frac{5}{8}$	0.625
$\frac{6}{8}$	0.75
$\frac{7}{8}$	0.875
$\frac{8}{8}$	1
$\frac{9}{8}$	1.125
$\frac{10}{8}$	1.25

Denominator 9:

Fraction	Decimal
$\frac{1}{9}$	$0.\overline{111}$
$\frac{2}{9}$	$0.\overline{222}$
$\frac{3}{9}$	$0.\overline{333}$
$\frac{4}{9}$	$0.\overline{444}$
$\frac{5}{9}$	$0.\overline{555}$
$\frac{6}{9}$	$0.\overline{666}$
$\frac{7}{9}$	$0.\overline{777}$
$\frac{8}{9}$	$0.\overline{888}$
$\frac{9}{9}$	1
$\frac{10}{9}$	$1.\overline{111}$

Denominator 11

Fraction	Decimal
$\frac{1}{11} = \frac{9}{99}$	$0.\overline{09}$
$\frac{2}{11}$	$0.\overline{18}$
$\frac{3}{11}$	$0.\overline{27}$
$\frac{4}{11}$	$0.\overline{36}$
$\frac{5}{11}$	$0.\overline{45}$
$\frac{6}{11}$	$0.\overline{54}$
$\frac{7}{11}$	$0.\overline{63}$
$\frac{8}{11}$	$0.\overline{72}$.
$\frac{9}{11}$	$0.\overline{81}$
$\frac{10}{11}$	$0.\overline{90}$

2. What do you notice about all fractions with 3, 6, or 9 as a denominator? What patterns did you notice? Explain?

$\frac{1}{3} = 0.\overline{3}$ $\frac{1}{9} = \frac{1}{3} \times \frac{1}{3} = 0.\overline{1}$

$\frac{1}{6} = \frac{1}{3} \times \frac{1}{2} = 0.1\overline{6}$

3. What do you notice about all fractions with 2, 4, or 8 as a denominator? What patterns did you notice? Explain?

$\frac{1}{2} = 0.5$ $\frac{1}{8} = \frac{1}{4} \times \frac{1}{2} = 0.125$

$\frac{1}{4} = \frac{1}{2} \times \frac{1}{2} = 0.25$

- No repeating digits

4. What do you notice about all fractions with 7 as a denominator? What patterns did you notice? Explain?

Has digits 1 4 2 8 5 7 in the same order

5. What patterns do you think there will be for all fractions with 11, 99, 999 or 9999 as a denominator? Explain and justify your answer.

Has same amount of repeating digits as the numbers of the denominator

6. Evaluate each of the following expressions to atleast three decimal places without using a calculator:

a) $\frac{35}{45} + \frac{14}{18}$

$= \frac{7}{9} + \frac{7}{9}$

$= \frac{14}{9}$ 1.555

b) $\frac{1}{3} \times \frac{6}{10} \times \frac{15}{21} \times \frac{28}{36} \times \frac{45}{55}$

0.09

c) $0.007 \div 0.35$

$\frac{7}{1000} \div \frac{7}{20} = \frac{1}{50}$

0.02

d) $(0.375)^3 \times \left(\frac{2}{3}\right)^2$

$\frac{3}{8} \times \frac{3}{8} \times \frac{3}{8} \times \frac{2}{3} \times \frac{2}{3} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ $= 0.375 \times 0.125 \times 0.5$

≈ 0.023

e) $(0.\overline{3} - 0.25) \times 1.2$

$\left(\frac{1}{3} - \frac{1}{4}\right) \times \frac{6}{5} = \frac{1}{12} \times \frac{6}{5}$

0.1

f) $0.\overline{2} \times 0.5 - 0.\overline{6}$

$\frac{2}{9} \times \frac{1}{2} - \frac{2}{3} = -\frac{5}{9}$

-0.555

g) $0.\overline{7} \times 0.\overline{857142} - 0.4$

$\frac{7}{9} \times \frac{6}{7} - \frac{2}{5} = \frac{2}{3} - \frac{2}{5}$

$0.2\overline{6}$

h) $\frac{2}{7} \times 0.\overline{55} \times 1.4 \div \frac{5}{4}$

$\frac{2}{7} \times \frac{5}{9} \times \frac{7}{5} \times \frac{4}{5} = 0.\overline{22} \times 0.8$

$0.1\overline{77}$

7. Simplify each of the following as a single fraction to lowest terms:

a) $(0.\overline{2})(0.\overline{3})$

$\frac{2}{9} \times \frac{1}{3}$

$\frac{2}{27}$

b) $(0.\overline{13})(0.\overline{5})$

$\frac{13}{99} \times \frac{5}{9}$

$\frac{65}{891}$

c) $(0.875) \times (0.\overline{4}) \times (0.75)$

$\frac{7}{8} \times \frac{4}{9} \times \frac{3}{4}$

$\frac{7}{24}$

d) $0.\overline{12} \times 0.375 - 0.\overline{2}$

$\frac{12}{99} \times \frac{3}{8} - \frac{2}{9} = \frac{1}{22} - \frac{2}{9}$

$9 - 44$

$\frac{35}{198}$

e) $0.\overline{63} \div 1.75$

$\frac{63}{99} \div \frac{7}{4}$

$\frac{7}{11} \times \frac{4}{7}$

$\frac{4}{11}$

f) $0.\overline{44} \times 2.25 - 0.\overline{5}$

$\frac{4}{9} \times \frac{9}{4} - \frac{5}{9}$

$\frac{4}{9}$

g) $(2.\overline{3}) \times (0.125) \times 0.875$

$\frac{21}{9} \times \frac{1}{8} \times \frac{7}{8}$

$\frac{147}{576}$

h) $0.\overline{285714} \times 1.75 \div 1.25$

$\frac{2}{7} \times \frac{7}{4} \div \frac{5}{4}$

$\frac{2}{5}$

8. What is the digit of the 100th decimal place for $\frac{1}{7}$? 8.

$\frac{1}{7} = 0.\overline{142857}$

$100 \div 6 = 16R4$

9. What is the digit of the 200th decimal place for $\frac{1}{13}$? 7.

$\frac{1}{13} = 0.\overline{076923}$

$200 \div 6 = 33R2$

10. Given that the first 8 digits for the decimal representation of $\frac{1}{17}$ is 0.05882341, derive the next 8 digits. 94117647.

16 repeating digits.

$\frac{1}{17} = 0.\overline{0588235294117647}$

$17 - 1 = 16$

11. How many repeating digits does the decimal representation of fraction $\frac{1}{19}$ have? Write them down. 18.

$19 - 1 = 18.$

12. Simplify each of the following expressions without using a calculator:

a) $0.\overline{428571} - 0.\overline{285714}$

$= \frac{3}{7} - \frac{2}{7}$

$\frac{1}{7}$

b) $\frac{0.875 - 0.125}{0.25}$

$= \frac{\frac{7}{8} - \frac{1}{8}}{\frac{1}{4}} = \frac{3}{4} \cdot 4$

3

c) $\frac{0.15 - 0.\overline{3}}{0.\overline{5}}$

$= \frac{\frac{3}{20} - \frac{1}{3}}{\frac{5}{9}} = -\frac{11}{60} \cdot \frac{9}{5}$

$-\frac{33}{100} (-0.33)$

d) $\left(0.\overline{2}\right)^2 \times \left(0.\overline{3}\right)^2$

$\left(\frac{2}{9}\right)^2 \times \left(\frac{1}{3}\right)^2$

$= \frac{4}{81} \times \frac{1}{9}$

$= \frac{4}{729}$

e) $\frac{\left(0.\overline{18}\right) - \left(0.\overline{3}\right)}{\left(0.\overline{27}\right)}$

$= \frac{\frac{2}{11} - \frac{1}{3}}{\frac{3}{11}}$

$= -\frac{5}{33} \cdot \frac{11}{3}$

$-\frac{5}{9} (-0.\overline{5})$

f) $\frac{0.\overline{72} - 0.\overline{45}}{0.75}$

$= \frac{\frac{8}{11} - \frac{5}{11}}{\frac{3}{4}}$

$= \frac{3}{11} \cdot \frac{4}{3}$

$\frac{4}{11}$

g) $\frac{0.375 + (0.8)^{-1} - 0.25}{(0.25)^2}$

$= \frac{\frac{3}{8} + \frac{5}{4} - \frac{1}{4}}{\frac{1}{16}}$

$= -\frac{5}{8} \cdot 16$

$-10.$

h) $\frac{0.3 - 0.15}{0.\overline{2} \times 0.\overline{3}}$

$\frac{0.15}{\frac{2}{9} \times \frac{1}{3}}$

$= \frac{5}{33} \cdot \frac{27}{2}$

$\frac{45}{22}$

13. Convert the following decimal representations to a mixed fraction:

a) 0.00353

$\frac{353}{10000}$

b) $14.\overline{285714}$ $\frac{2}{7}$

$14\frac{2}{7}$

c) $3.23232323...$

$3\frac{23}{99}$

d) $-4.151515...$

$-4\frac{3}{20}$

e) $0.\overline{255}$ $= \frac{255}{999} = \frac{85}{333}$

$\frac{85}{333}$

f) $0.\overline{0255}$ $= \frac{255}{9999} = \frac{85}{3333}$

$\frac{85}{3333}$

g) $0.0\overline{255}$ $\frac{255}{9990}$

$\frac{17}{666}$

h) $0.0\overline{0255}$ $\frac{255}{99990}$

$\frac{17}{6666}$

i) $0.0\overline{00671}$

$\frac{671}{99999}$

j) $0.000\overline{3817}$ $\frac{3817}{9999000}$

$\frac{347}{909000}$

k) ~~$0.1212212221....$~~

l) $0.\overline{38501927}$

$\frac{38501927}{99999999}$

14. Let $x = 0.7181818...$, where the digits '18' repeat. When x is expressed as a fraction in lowest terms, then its denominator exceeds its numerator by:

(a) 18 (b) 31 (c) 93 (d) 141 (e) 279

$0.7\overline{18} = \frac{718-7}{990} = \frac{711}{990} = \frac{79}{110}$

$110 - 79 = \boxed{31}$ //

Name: Claire Date: 9.6

1.2 Irrational and Rational Numbers Worksheet:

1. Indicate which of the following are Irrational Numbers. If it is not a rational number, please simplify the expression:

$\sqrt{12}$ $\sqrt{16}$ 1.13 $\sqrt{\sqrt{81}}$ $5.\overline{32}$ 1.875132 $\sqrt[3]{11}$ $\sqrt[5]{32}$ $\sqrt[3]{27}$ $\sqrt{\frac{25}{16}}$

$2\sqrt{3}$. 4 3 2. 3. $\frac{5}{4}$

$-8.1313313331...$ $2\frac{5}{6}$ $\left(\left(\sqrt{5}\right)\sqrt{20}\right)$ $\frac{\sqrt{10}}{\sqrt{20}}$ $\sqrt[3]{125}$ $\sqrt[5]{31+1}$ $85.123456....$

10. $\frac{\sqrt{2}}{2}$ 5.

2. Use a number line to order these numbers from the least to the greatest, then place them on a number line

$5\sqrt{2}$. 10 $5\sqrt[4]{4}$

a) $\sqrt{50}, \sqrt[2]{100}, \sqrt[3]{500}, \sqrt[4]{1000}$

$\sqrt[3]{500}$ $\sqrt[4]{1000}$ $5\sqrt{2}$. $\sqrt[2]{100}$

b) $\sqrt[4]{250}, \sqrt[2]{300}, \sqrt[3]{-180}, \sqrt{89}$ ≈ 9.

≈ 5 $10\sqrt{3}$. ≈ -6.

$\sqrt[3]{180}$ $\sqrt[4]{250}$ $\sqrt{89}$ $\sqrt[2]{300}$.

c) $\sqrt{499}, 5\sqrt{23}, \left(\sqrt[3]{88}\right)^2, \left(12\sqrt[4]{20}\right)$

≈ 22 ≈ 25 ≈ 16 ≈ 24

$\sqrt[3]{88}$ $\sqrt{499}$ $5\sqrt{23}$ $12\sqrt[4]{20}$

3. Find the decimal representation for following. Indicate any patterns that you see:

$\frac{1}{7} = 0.\overline{142857}$ $\frac{2}{7} = 0.\overline{285714}$ $\frac{3}{7} = 0.\overline{428571}$ $\frac{4}{7} = 0.\overline{571428}$

$\frac{5}{7} = 0.\overline{714285}$ $\frac{6}{7} = 0.\overline{857142}$

$\frac{9}{99}$. $\frac{1}{11} = 0.\overline{09}$ $\frac{2}{11} = 0.\overline{18}$ $\frac{3}{11} = 0.\overline{27}$ $\frac{4}{11} = 0.\overline{36}$

$\frac{1}{13} = 0.\overline{076923}$ $\frac{2}{13} = 0.\overline{153846}$ $\frac{3}{13} = 0.\overline{230769}$ $\frac{4}{13} = 0.\overline{307692}$.

0
3. 7
2
9 6.

$\frac{5}{13} = 0.\overline{384615}$ $\frac{6}{13} = 0.\overline{461538}$ $\frac{7}{13} = 0.\overline{538461}$ $\frac{8}{13} = 0.\overline{615384}$

$\frac{9}{13} = 0.\overline{692307}$ $\frac{10}{13} = 0.\overline{769230}$ $\frac{11}{13} = 0.\overline{846153}$ $\frac{12}{13} = 0.\overline{923076}$

b) What patterns do you notice about fractions with a denominator of 13? How many repeating digits are there?

3. 0 7 6 9 2 or 6. 1 5 3 8 4

6 repeating digits.

4. Which of the following statements are true?

i) All natural numbers are integers. True.

ii) All integers are rational numbers. True

iii) All whole numbers (0, 1, 2, 3) are natural numbers (1, 2, 3) False.

iv) All irrational numbers are roots π. False.

v) Some rational numbers are natural numbers True.

vi) The sum a rational number and an irrational number will be rational False.

vii) The product of a rational number and an irrational number will be irrational True

viii) Zero is a whole number but not a natural number True.

ix) $\sqrt{23}$ is a real number but not a rational number True.

5. Write a number that is

a) a rational number but not an integer $\frac{1}{5}$

b) a whole number but not a natural number 0.

c) write a number that is not a real number $\sqrt{-2}$

6. The first 9 digits in the decimal representation of $\frac{5}{17}$ is: 0.294117647. Use this information to find the remaining digits in the repeating pattern.

$0.\overline{2941176470588235}$

b) Without using a calculator, use the information previously to find the decimal representation of $\frac{9}{17}$

$0.\overline{5294117647058823}$

c) Use the prior information to find the decimal representation of all fractions with a denominator of 17 from $\frac{1}{17}$ to $\frac{16}{17}$

$\frac{1}{17} = 0.\overline{0588235294117647}$

$\frac{2}{17} = 0.\overline{1176470588235294}$

$\frac{3}{17} = 0.\overline{1764705882352941}$

$\frac{4}{17} = 0.\overline{2352941176470588}$

$\frac{5}{17} = 0.\overline{2941176470588235}$

$\frac{6}{17} = 0.\overline{352941176470588}$

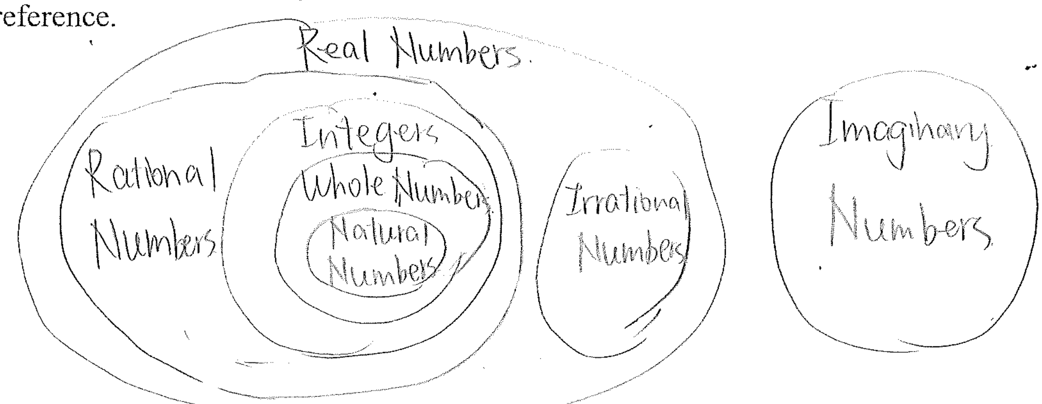
$\frac{7}{17} = 0.\overline{4117647058823529}$

$\frac{8}{17} = 0.\overline{470588235294117}6$

$\frac{9}{17} = 0.\overline{5294117647058823}$

$\frac{10}{17} = 0.\overline{5882352941176470}$

7. Create your own venn diagram to show how the following numbers are related: Real Numbers, Imaginary Numbers, Rational Numbers, Irrational Numbers, Whole Numbers, Integers, Natural Numbers. You may use the internet for reference.



8. Evaluate each of the following without a calculator. Show all your work and justify your solution:

i) $\sqrt[3]{5} \times \sqrt[3]{5} \times \sqrt[3]{5}$

$= \sqrt[3]{5^3}$

$= 5$

ii) $\sqrt[6]{25} \times \sqrt[6]{25} \times \sqrt[6]{25} \times \sqrt[6]{25} \times \sqrt[6]{25} \times \sqrt[6]{25}$

$= \sqrt[6]{25^6}$

$= 25$

iii) $\sqrt{2}\left(\sqrt{2} + \sqrt{2} + \sqrt{2}\right)$

$= \sqrt{2} \times 3\sqrt{2}$

$= 6$

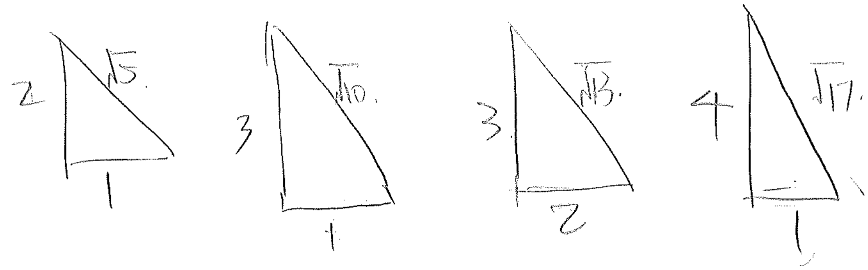
iv) $\sqrt{6 \times 5} \times \sqrt{45} \times \sqrt{24}$

$= \sqrt{3 \times 2 \times 5} \times \sqrt{3 \times 3 \times 5} \times \sqrt{2 \times 2 \times 3 \times 2}$

$= 2^2 \times 3^2 \times 5$

$= 180$

9. Lengths that are Irrational numbers can be created using right triangles with legs that are integers lengths with the Pythagorean Theorem: ($a^2 + b^2 = c^2$). For instance, the irrational number $\sqrt{2}$ can be made from a right triangle with legs of unit length 1 and 1. Create right triangles with legs of unit lengths to generate each of the following lengths: $\sqrt{5}, \sqrt{10}, \sqrt{13}, and \sqrt{17}$.



10. Suppose you know that $\sqrt[n]{\frac{a}{b}}$ is a rational number and that 'a' and 'b' have no common factors. What can you say about the prime factorizations of 'a' and 'b'?

a and b are perfect squares

11. The value of $0.\overline{1}+0.\overline{12}+0.\overline{123}$ is:

(A) $0.\overline{343}$ (B) $0.\overline{355}$ (C) $0.3\overline{5}$ (D) $0.\overline{355446}$ (E) $0.\overline{355445}$

$$\frac{1}{9}+\frac{12}{99}+\frac{123}{999}$$
$$=\frac{111111}{999999}+\frac{121212}{999999}+\frac{123123}{999999}=\frac{355446}{999999}=0.\overline{355446}$$

12. In the sequence of fractions $\frac{1}{1},\frac{2}{1},\frac{1}{2},\frac{3}{1},\frac{2}{2},\frac{1}{3},\frac{4}{1},\frac{3}{2},\frac{2}{3},\frac{1}{4},\frac{5}{1},\ldots$ $\frac{4}{2}$ $\frac{3}{3}$ $\frac{2}{4}$ fractions equivalent to any given fraction occur many times. For example, fractions equivalent to $\frac{1}{2}$ occur for the first two times in positions 3 and 14. In which position is the fifth occurrence of a fraction equivalent to $\frac{3}{7}$?

$\frac{3}{7}$ $\frac{6}{14}$ $\frac{9}{21}$ $\frac{12}{28}$ $\frac{15}{35}$

(A) 1207 (B) 1208 (C) 1209 (D) 1210 (E) 1211

$1225-(49-35)=1211$

$15+35-1=49$

$49(49+1)/2=1225$

13. Let $x=0.7181818\ldots$, where the digits '18' repeat. When x is expressed as a fraction in lowest terms, then its denominator exceeds its numerator by:

(a) 18 (b) 31 (c) 93 (d) 141 (e) 279

$0.7\overline{18}$

$=\frac{718-7}{990}$

$=\frac{711}{990}$

$=\frac{79}{110}$

$110-79=31$

Name: Claire Date: 9.10

Math 9H HW 1.3 Prime Factorization, Perfect Squares, and Cubes (Calculators not Allowed)

1. Find the Prime factorization for each of the following numbers:

a) 24

$24 = 2^3 \times 3$

d) 360

$360 = 2^3 \times 3^2 \times 5$

g) 9360

$9360 = 2^4 \times 3^2 \times 5 \times 13$

b) 17640

$17640 = 2^3 \cdot 3^2 \cdot 5 \cdot 7^2$

e) 9492

$9492 = 2^2 \cdot 3 \cdot 7 \cdot 113$

h) 9418409

$9418409 = 7 \times 11 \times 13 \times 97^2$

113

2. Indicate which of the following numbers are perfect squares, perfect cubes, or neither:

a) $N = 16 \times 3^3 \times 14$

$= 2^4 \times 3^3 \times 2 \times 7$

$= 2^5 \times 3^3 \times 7$ Neither

e) $N = 8 \times 25 \times 81$

$= 2^3 \times 5^2 \times 3^4$

Neither

b) $N = 4^3 \times 9^3$

$= 2^6 \times 3^6$

Perfect square & Perfect cube

f) $N = 7 \times 7^2 \times 7^4$

$= 7^7$

Neither

c) $N = 3 \times 4^3 \times 12$

$= 3^2 \times 4^4$

$= 3^2 \times 2^8$

Perfect square

g) $N = 27 \times 5^3 \times 135$

$= 3^3 \times 5^3 \times 3^3 \times 5$

$= 3^6 \times 5^4$

Perfect square

3. Simplify each of the following expressions without a calculator:

i) $\sqrt{12 \times 24}$

$= 12\sqrt{2}$

ii) $\sqrt{48 \times 243}$

$= \sqrt{3 \times 2^4 \times 3^5}$

$= 2^2 \times 3^3$

$= 108$

iii) $\sqrt{32 \times 45 \times 8}$

$= \sqrt{16^2 \times 3^2 \times 5}$

$48\sqrt{5}$

iv) $\sqrt{6 \times 9 \times 24}$

$= \sqrt{6 \times 3 \times 3 \times 6 \times 4}$

$= 36$

v) $\sqrt{252 \times 567}$

$= \sqrt{4 \times 9 \times 7 \times 7 \times 81}$

$= \sqrt{2^2 \times 27^2 \times 7^2}$

$= 378$

vi) $\sqrt{147 \times 27 \times 289}$

$= \sqrt{7^2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 17^2}$

$\sqrt{7^2 \times 9^2 \times 17^2} = 1071$

vii) $\sqrt[3]{432 \times 375}$

$= \sqrt[3]{2^4 \times 3^4 \times 5^3}$

$= 30\sqrt[3]{6}$

viii) $\sqrt[3]{6 \times 60 \times 75}$

$= \sqrt[3]{2^3 \times 3^3 \times 5^3}$

$= 30$

ix) $\sqrt[4]{252 \times 294 \times 42}$

$= \sqrt[4]{4 \times 9 \times 7 \times 2 \times 7^2 \times 3 \times 3 \times 2 \times 7}$

$= \sqrt[4]{2^4 \times 3^4 \times 7^4}$

$= 42$

4. Given that "N" is an integer, find the lowest value of N such that the following will be a positive integer:

a) $\sqrt{2^3 5^1 7^2 N}$

$N = 2\times5$

10

b) $\sqrt{4^2 7^2 5^2 N}$

$N = 1$.

1.

c) $\sqrt{3^4 5^3 12N}$

$2^2 3^5 \cdot 5^3$. $N = 3\times5$

15.

d) $\sqrt{38412N}$

$\sqrt{2^2\times3^2\times7\times97 \cdot N}$

$N = 7\times97$.

673

e) $\sqrt{13992N}$

$\sqrt{2^3\times3\times11\times53\times N}$

$N = 2\times3\times11\times53$.

2698

f) $\sqrt{664(N-1)}$

$\sqrt{2^3 \cdot 83(N-1)}$

$N-1 = 2\times83$.

167

g) $\sqrt[3]{2^3 11^3 49^2 N}$

$\sqrt[3]{2^3 11^3 7^4 N}$

$N = 7^2$

49.

h) $\sqrt[3]{6^5 5^2 15^2 N}$

$\sqrt[3]{2^5\times3^5\times5^2\times3^2\times5^2 N}$

$\sqrt[3]{2^5\times3^7\times5^4 N}$

$N = 2\times3^2\times5^2$

450.

i) $\sqrt[3]{3^4 5^3 28N}$

$\sqrt[3]{3^4\times2^2\times5^3\times7 N}$

$N = 3^2\times2\times7^2$.

882.

5. Given that N is an integer and $N \neq 0$, what is the lowest value of N so that "K" is
i) a perfect square ii) a perfect cube iii) Both a perfect square and perfect cube $(Given\ K \neq 0)$

a) $K = N\times3^3\times21 = N\times3^4\times7$.

i) $N = 7$.

ii) $N = 3^2\times7^2 = 441$

iii) $N = 3^2\times7^5 = 151263$.

b) $K = N\times3^3\times5^5$

i) $N = 3\times5 = 15$.

ii) $N = 5$

iii) $N = 3^3\times5 = 135$.

c) $K = N\times75\times169$

$= N\times3\times5^2\times13^2$

i) $N = 3$.

ii) $N = 3^2\times5\times13 = 585$

iii) $N = 3^5\times5^4\times13^4$

d) $K = (N-1)\times7^7\times121 = (N-1)\times7^7\times11^2$

i) $N-1 = 7 \therefore N = 8$.

ii) $N-1 = 7^2\times11 = 539 \therefore N = 540$.

iii) $N = 7^5\times11^4 + 1$

e) $K = N^2 + N$

$= N(N+1)$.

i) Impossible. $N \neq N+1$

ii). $N+1 = N^2$. $0 = N^2 - N - 1$ Impossible.

iii). Impossible

f) $K = (3N-24)(N-28)$

i) $3N-24 = N-28$

$2N = -4$

$N = -2$

ii). $3N-24 = (N-28)^2$

$3N-24 = N^2-56N+784$.

$N^2 - 59N + 808 = 0$

$\because$ N is not an integer.

$\therefore$ Impossible.

iii) Impossible.

6. Find the value of "N" in each equation:

$18{,}000 = N\times2^3\times5^3\times6$

$2\times3^2\times2^3\times5^3 = N\times2^3\times5^3\times2\times3$.

$2^4\times3^2\times5^3 = N\times2^4\times3\times5^3$

$N = 3$.

b) $80{,}640 = N\times2^5\times12\times14$

$2^8\times3^2\times5\times7 = N\times2^5\times2^2\times3\times2\times7$.

$N = 15$.

7. For what integer $n>0$ does $(100k)^2 \times (100k)^2 = n^2k^4$ for all values of k?

$100^2 \times k^2 \times 100^2 \times k^2 = n^2 k^4$

$(100^2)^2 = n^2$

$n = 100^2$

$n = \boxed{10000}$

8. Let a, b, c, d, and e be distinct integers such that: $(6-a)(6-b)(6-c)(6-d)(6-e)=45$. What is the value of $a+b+c+d+e$?

$45 = 3^2 \times 5 \rightarrow 1 \times -1 \times -3 \times 3 \times 5 = 45$

$\begin{cases} 6-a=1 \\ 6-b=-1 \\ 6-c=-3 \\ 6-d=3 \\ 6-e=5 \end{cases} \therefore \begin{cases} a=5 \\ b=7 \\ c=9 \\ d=3 \\ e=1 \end{cases}$

$a+b+c+d+e$
$=5+7+9+3+1$
$=\boxed{25}$

9. The number 1000 has 16 positive integral divisors. How many positive divisors does the number 3000 have?

$3000 = 2^3 \cdot 3^1 \cdot 5^3$

$(3+1) \times (1+1) \times (3+1)$
$= 4 \times 2 \times 4$
$= \boxed{32}$

10. Solve for 'n' $(n-23)! \times 23! = (n-32)!32!$

$\because 32! \times 23! = 23! \times 32!$

$\therefore \begin{cases} n-23=32 \\ n-32=23 \end{cases}$

$\therefore n=55$

11. If "N" is the product of three different primes, then its least possible value is $2\times3\times5=30$. If $N<100$, what is N's largest possible value?

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37

Way ① $100 = 2^2 \times 5^2$
$99 = 9 \times 11$
$98 = 2 \times 49$
97
$96 = 3 \times 2^5$
$95 = 5 \times 19$
$94 = 2 \times 47$
$93 = 3 \times 31$
$92 = 4 \times 23$
$91 = 7 \times 13$
$90 = 3^2 \cdot 2 \cdot 5$
89
$88 = 8 \times 11$
87
$86 = 2 \times 43$
$85 = 5 \times 17$
$84 = 7 \times 12$
83
$82 = 2 \times 41$
$81 = 9^2$
$80 = 2^4 \cdot 5$
79
$78 = 2 \times 3 \times 13$

Way ② $2 \times 5 \times 7 = \cancel{70}$
$2 \times 3 \times 13 = \underline{78}$

12. Challenge: Find the smallest value for N, such that N>1, so that the following expression is a perfect cube:

$5N^3 + N^2 + 15N$
$= N(5N^2+N+15)$

① $5N^2+N+15 = N^2$
$4N^2+N+15 = 0$
$N = \frac{-1 \pm \sqrt{1-240}}{8}$ X

② $N\underbrace{(5N^2+N+15)}_{A^3N^2} = A^3N^3$
$5N^2+N+15 = A^3N^2$
1) $A=2$
$5N^2+N+15 = 8N^2$
$-3N^2+N+15 = 0$

Square Roots and Mixed Radicals

Name: Claire Li Date: 9.12.

1. Simplify each of the following Radicals:

i) $\sqrt[3]{8}$ 2 *ii)* $\sqrt[3]{-27}$ -3 *iii)* $\sqrt[4]{81}$ 3. *iv)* $\sqrt[5]{-32}$ -2. *v)* $\sqrt[3]{0.001}$ 0.1

vi) $\sqrt[4]{64}$ $2\sqrt[4]{4}$. *vii)* $\sqrt[4]{256}$ 4 *viii)* $\sqrt[3]{-1000}$ -10. *ix)* $\sqrt[3]{7^6}$ 49. *x)* $\sqrt[3]{216^4}$ 1296.

2. Simplify each radical to an integer if possible:

i) $\sqrt[3]{\sqrt{729}}$ 3 *ii)* $\sqrt[3]{10-\sqrt{4}}$ 2. *iii)* $\sqrt{4+9+36}$ 7. *iv)* $\sqrt[3]{27+64+125}$ 6.

v) $\sqrt{64}+\sqrt[3]{64}$ 12.

vi) $\sqrt{25}+\sqrt[3]{27}-\sqrt{\sqrt[3]{729}}$ $=5+3-3$ 5.

vii) $5\sqrt{9}-4\sqrt[3]{-27}$ $=15+4\times3$. 27.

viii) $\sqrt[3]{226-\sqrt{100}}$ $\sqrt[3]{226-10}$ 6.

3. Express each of the following as a Mixed Radical

i) $\sqrt{18}$ $3\sqrt{2}$ *ii)* $\sqrt{72}$ $6\sqrt{2}$ *iii)* $\sqrt{105}$ $\sqrt{105}$ *iv)* $\sqrt{126}$ $3\sqrt{14}$ *v)* $\sqrt{96}$ $4\sqrt{6}$.

vi) $\sqrt{147}$ $7\sqrt{3}$ *vii)* $\sqrt{54}$ $3\sqrt{6}$. *viii)* $\sqrt{76}$ $2\sqrt{19}$. *ix)* $\sqrt{180}$ $6\sqrt{5}$ *x)* $5\sqrt{18}$ $15\sqrt{2}$

4. Simplify each of the following into mixed radicals

i) $\sqrt{24}\times\sqrt{18}\times3\sqrt{15}$
$=3\sqrt{3\times2^3\times3^2\times2\times3\times5}$
$108\sqrt{5}$

ii) $4\sqrt{8}\times3\sqrt{6}\times\left(-7\sqrt{3}\right)$
$=-84\times12$
$=-1008$.

iii) $5\sqrt{18}\times4\sqrt{8}\times\left(-3\sqrt{32}\right)$
$=-60\sqrt{3^2\times2\cdot2^3\times2^5}$
$-2880\sqrt{2}$.

iv) $\sqrt{3m}\times\sqrt{5m}$
$\sqrt{15}m$

v) $11\sqrt{6b}\times4\sqrt{3ab}\times3\sqrt{8a}$
$=11\times4\times3\times12\times ab$
$1584ab$.

vi) $5\sqrt{6ab}\times4\sqrt{12a^2b}\times3\sqrt{9b}$
$=60\sqrt{3\times2\times2^2\times3\times3^2a^3b^3}$
$1080a\sqrt{2ab}$

Arrange the radicals from the least to the greatest

i) $9\sqrt{2},\ 2\sqrt{6},\ 8\sqrt{3},\ 2\sqrt{4},\ -3\sqrt{5}$ $\quad$ $-3\sqrt{5} < 2\sqrt{4} < 2\sqrt{6} < 9\sqrt{2} < 8\sqrt{3}$.

$\sqrt{162}$ $\quad$ $\sqrt{192}$

ii) $-6\sqrt{2},\ -3\sqrt{7},\ -2\sqrt{17},\ -4\sqrt{5},\ 2\sqrt{21},\ -5\sqrt{3}$ $\quad$ $-4\sqrt{5} < -5\sqrt{3} < 6\sqrt{2} < -2\sqrt{17} < -\sqrt{68} < \sqrt{84}$

$-\sqrt{72}\ \ -\sqrt{63}\ \ -\sqrt{68}\ \ -\sqrt{80}\ \ \sqrt{84}\ \ -\sqrt{75}$.

iii) $\sqrt{500},\ 5\sqrt{21},\ \left(\sqrt[3]{99}\right)^2,\ 10\sqrt[4]{20}$ $\quad$ $10\sqrt[4]{20} < \left(\sqrt[3]{99}\right)^2 < \sqrt{500} < 5\sqrt{21}$

$\sqrt{500}$ $\quad$ $\sqrt{525}$ $\quad$ $\sqrt[3]{9801}$ $\quad$ $\sqrt[4]{20000}$

5. Given that $\sqrt{2} = 1.4142$, determine a decimal representation for $\sqrt{20}$ and $\sqrt{20,000}$ without a calculator

$\sqrt{20} =$ 4.66686 $\qquad$ $\sqrt{20,000} =$ 141.42

6. Indicate whether if the following statements are either true or false: $\quad$ $\sqrt{a^2b} + \sqrt{ab^2} = \sqrt{ab}\left(\sqrt{a} + \sqrt{b}\right)$.

a) $\sqrt{8} + \sqrt{4} = \sqrt{12}$ $\quad$ False

d) $\sqrt{a} + \sqrt{b} = \sqrt{a+b}$ $\quad$ False.

g) $a\sqrt{b} + b\sqrt{a} = ab\sqrt{ab}$ $\quad$ False.

b) $3\sqrt{3} \le \sqrt[3]{3}$ $\quad$ False.

e) $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ $\quad$ True.

h) $a\sqrt{c} + b\sqrt{c} = (a+b)\sqrt{c}$ $\quad$ True.

c) $2\left(4\sqrt{7}\right) = 8\sqrt{14}$ $\quad$ False

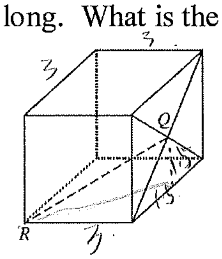
f) $a \times \sqrt{b} = a\sqrt{b}$ $\quad$ True.

i) $a\sqrt{c} \times b\sqrt{c} = (a \times b) \times c$ $\quad$ True

j) Every mixed radical can be expressed as an entire radical? True.

k) Every entire radical can be expressed as a mixed radical? False.

7. Point Q is the point of intersection of the diagonals of one face of a cube whose edges have length 3 units long. What is the length of QR?



$\sqrt{9 + \frac{9}{4}} = \sqrt{\frac{45}{4}}$ $\qquad$ $\sqrt{\frac{45}{4} + \frac{9}{4}} = \sqrt{\frac{54}{4}} = \frac{3\sqrt{6}}{2}$

$QR = \frac{3\sqrt{6}}{2}$.

8. Given that: $\sqrt{x\sqrt{x\sqrt{x\sqrt{x\sqrt{x^a}}}}} = x^3$, what is the value of 'a'? $\quad$ $a =$ 34

$x\sqrt{x\sqrt{x\sqrt{x\sqrt{x^a}}}} = x^6$.

$x\sqrt{x\sqrt{x\sqrt{x^a}}} = x^{10}$.

$x\sqrt{x^a} = x^{18}$

$x^a = x^{34}$

9. If $4=\sqrt{x+\sqrt{x+\sqrt{x+\sqrt{x+\ldots\ldots}}}}$, then find the value of "x" — 12

$16 = x+\sqrt{x+\sqrt{x+\sqrt{x+\cdots}}}$ $\qquad 4 = 16 - x$

$16 - x = \sqrt{x+\sqrt{x+\sqrt{x+\cdots}}}$ $\qquad x = 12$

10. Given that $\sqrt{5-2\sqrt{6}}=\sqrt{a}-\sqrt{b}$, find the values of 'a' and 'b'. — 2, 3.

$5-2\sqrt{6} = a+b-2\sqrt{ab}$

$\begin{cases} a+b=5 \\ ab=6 \end{cases}$ $\begin{cases} a=2 \\ b=3 \end{cases}$ or $\begin{cases} a=3 \\ b=2 \end{cases}$

11. Simplify: $\sqrt{2^5+2^5+2^5+2^5+2^5+2^5+2^5+2^5}$ without using a calculator. — $2^4(16)$

$\sqrt{8\times2^5}$

$=\sqrt{2^3\times2^5} = 2^4$

12. How many whole numbers are there between $\sqrt{11}$ and $\sqrt{111}$? — 7.

$3<\sqrt{11}<4$ $\qquad 4, 5, 6, 7, 8, 9, 10.$

$10<\sqrt{111}<11$

13. What is the value of $\sqrt{(24)\times(60)\times(105)\times(42)}$? (No Calculator) — 2520.

$\sqrt{4\times6\times6\times2\times5\times5\times3\times7\times6\times7}$

$=2\times6\times6\times5\times7 = 2520$

14. If $i^2=-1$, then what is the value of i^{253}? — i

$i^4 = i^2\cdot i^2 = 1$ $\qquad i^{252}\cdot i$

$i^6 = -1$ $\qquad 1\cdot i = i$

15. The numbers 'a' and 'b' are consecutive positive integers, and $a<\sqrt{200}<b$.
What is the value of the product of ab? — 210.

$14<\sqrt{200}<15$

$14\times15 = 7\times3\times10$

16. Combine the following into a single radical;

a) $\left(\sqrt[3]{x^2}\right)\left(\sqrt[3]{x}\right)$ — x

b) $\left(\sqrt[3]{x^5}\right)\left(\sqrt[3]{x}\right)$ — x^2

c) $\left(a\sqrt{b^3}\right)\left(b\sqrt{a}\right)$ — $\sqrt{a^3b^5}$

d) $\sqrt{x^4}\left(\sqrt[3]{x}\right)$ — $\sqrt[5]{x^1}$

e) $\sqrt{4x}+\sqrt{9x}+\sqrt{121x}$

$=2\sqrt{x}+3\sqrt{x}+11\sqrt{x}$

$=16\sqrt{x}$ — $\sqrt{256x}$

f) $\sqrt[3]{x\sqrt{x^3}}$ — $\sqrt[6]{x^5}$

d) $x\left(\sqrt[3]{x\sqrt{x^3}}\right)$

$=\sqrt[3]{x^4\sqrt{x^3}} = \sqrt[3]{\sqrt{x^{11}}}$ — $\sqrt[6]{x^{11}}$

e) $x^4\sqrt{x^3\sqrt{x^2\sqrt{x}}}$

$=\sqrt{x^{11}\sqrt{x^2\sqrt{x}}} = \sqrt{\sqrt{x^{24}\sqrt{x}}} = \sqrt{\sqrt{\sqrt{x^{49}}}}$ — $\sqrt[8]{x^{49}}$

f) $\sqrt[3]{\left(x^2\right)\sqrt[3]{\left(x^2\right)\sqrt[3]{x}}}$ $\qquad \left(x^2\cdot\left(x^2\cdot x^{\frac{1}{3}}\right)^{\frac{1}{3}}\right)^{\frac{1}{3}}$

$=\sqrt[3]{\sqrt[3]{x^8\sqrt[3]{x}}} = \sqrt[3]{\sqrt[3]{\sqrt[3]{x^{25}}}}$ — $\sqrt[27]{x^{25}}$

Proof of divisibility rule of 7.

$$N = \underbrace{\quad a \quad}_{\text{other digits}} \underbrace{b}_{\text{unit's digit}}$$

$N = 10a + b$

if $a - 2b = 7k \ (k = 0, 1, 2, 3 \ldots)$

$10a - 20b = 7k$

$10a = 7k + 20b$

$N \cdot (10a + b) = 7k + 21b$ — divisible by 7.

$\therefore$ N is divisible by 7.

Proof of divisibility rule of 13.

$$N = \underbrace{\quad a \quad}_{\text{other digits}} \underbrace{b}_{\text{unit's digit}}$$

$N = 10a + b$

if $a - 9b = 13k \ (k = 0, 1, 2, 3 \ldots)$

$10a - 90b = 13k$

$10a = 13k + 90b$

$N \cdot (10a + b) = 13k + 91b$

divisible by 7.

$\therefore$ N is divisible by 13.

Math 9 Enriched

1.5 Divisibility

Name Claire Date 9.16.

1. Which number is *not* divisible by 3?

   582, 441, or 782

   V V

2. How many of the first 50 counting numbers are divisible by both 2 and 3?

   $50 \div (2\times3)$

   $= 50 \div 6$

   $= 8$

   8.

3. What value can a have to make $\underline{a74a}$ divisible by 36?

   9 4.

   $a = 0, 4, 8$

   $4+7+4+4 = 19$ X.

   $8+7+4+8 = 27$ V.

   $a = \boxed{8}$

4. If the three-digit number $\underline{56u}$ is divisible by 6, what is u?

   2 3.

   $u = 1, 4, 7.$

   $u = \boxed{4}$

5. What digit can replace k in the number $\underline{9k73k0}$ so that the number will be divisible by 60?

   3 4 5.

   $9+7+3+2k =$ multiple of 3

   $19+2k = 21, 27, 33.$

   $k = 1, 4, 7.$

   k0 is divisible by 4

   $k=4$, 40 V. $\therefore k = \boxed{4}$

6. What is the largest three-digit number divisible by 3, 4, 5, and 6?

   3, 4, 5.

   $3\times4\times5 = 60$

   $1000 \div 60 = 16 R 40.$

   $60\times16 = \boxed{960.}$

7. $\underline{abab}$ is a four-digit number with $a \neq 0$. What is the largest prime number by which $\underline{abab}$ must be divisible?

$abab = ab \times 101$

It must be divisible by $\boxed{101}$ //.

8. What percent of the odd numbers between 0 and 100 are multiples of 3?

$100 \div 3 = 33 R 1$

$100 \div (3 \times 2) = 16 R 4$

$33 - 16 = 17$

$\frac{17}{50} = 34\%$

9. If the three-digit number $\underline{2d2}$ is divisible by 7, what is d?

$\underline{2d} - 2 \times 2 = 7k (k = 0, 1, 2, 3 \cdots)$

$\underline{2d} - 4 = 21$

$\underline{2d} = 25$. $\therefore d = \boxed{5}$ //.

10. The five-digit number $\underline{6a47b}$ is divisible by 18. What is the smallest possible value of $a - b$?

$b = 0, 2, 4, 6, 8$.

$2 \quad 9$

$6 + a + 4 + 7 + b = 9k (k = 0, 1, 2, 3 \cdots)$

$17 + a + b = 18$ or 27!

① $17 + a + b = 18$.

$\begin{cases} a = 1 \\ b = 0 \end{cases}$ $a - b = 1$.

② $17 + a + b = 27$

$\begin{cases} a = 6 \\ b = 4 \end{cases}$ $a - b = 2$.

$\boxed{(a - b)_{min} = 1}$ //.

11. Find distinct digits A and B such that $\underline{A47B}$ is as large as possible and divisible by 36. Name the number.

$\underline{7B}$ is divisible by 4.

$\therefore B = 2, 6$

$A + 4 + 7 + B = 18, 27$.

① $B = 2, A = 5$ ② $B = 6, A = 1$

$4 \quad 9$.

$\boxed{5472}$ //.

12. Arrange the four digits 1, 2, 3 and 7 so that the four-digit number you get is a multiple of 8.

$\begin{cases} 3\ 7\ 1\ 2 \\ 7\ \underbrace{3\ 1\ 2}_{\text{multiple of 8}} \end{cases}$

13. Find the smallest value of $a + b$ for which the sum $\underline{4a5} + \underline{3b7}$ is a multiple of 9.

$\underline{4a5} + \underline{3b7}$

$= 400 + 10a + 5 + 300 + 10b + 7$

$= 712 + 10(a + b)$

$= \underbrace{[711 + 9(a + b)]}_{\text{divisible by 9}} + (1 + a + b)$

$1 + a + b = 9$

$a + b = \boxed{8}$ //.

14. Find the least possible value of digit d so that $\underline{437{,}d03}$ is divisible by 9.

$4+3+7+d+3=9k\ (k=0,1,2,3\cdots)$

$17+d=18.$

$d=\boxed{1}$ //.

15. For what digit a in the hundreds position will the six-digit number 907a32 be divisible by 33?

3 11.

$9+7+3-a-2=11k\ (k=0,1,2,3\cdots)$

$17-a=11.$

$a=\boxed{6.}$ // $\to$ $9+7+3+6+2=27$ ✓

16. How many positive integers less than 400 are divisible by both 7 and 11?

$400\div(7\times11)$

$=400\div77.$

$=5\ R\ 15.$

$\boxed{5}$ //.

17. For what digit n is the five-digit number $3n85n$ divisible by 6?

2 3.

$n=0,2,4,6,8.$

$3+8+5+2n=3k\ (k=0,1,2,3\cdots).$

$16+2n=18,24\cdots$

$n=1,4,7.$

$n=4.$

18. The six-digit number $3730n5$, with tens digit n, is divisible by 21. What is the value of the digit n?

$3+7+3+5+n=3k\ (k=0,1,2,3\cdots)$ 3 7.

$18+n=18,21,24\cdots$

$n=0,3,6,9.$

$\because$ 372400 is divisible by 7.

$3730n5=372400+\underline{6n5}$

① $n=0$ ~~605~~. ② $n=3$ ~~635~~. ③ $n=6.$ 665 ✓. ④ $n=9.$ ~~695~~.

$n=\boxed{6}$ //

19. For what value of n is the five-digit number $7n{,}933$ divisible by 33?

$7+9+3-3-n=11k\ (k=0,1,2,3\cdots)$ 3 11,

$16-n=11$

$n=\boxed{5.}$ // $\to$ $7+5+9+3+3=27.$ ✓.

20. Challenge: Find a nine digit number that is divisible by all the integers from 1 to 9

~~1~~, ~~2~~, ~~3~~, ~~4~~, 5, ~~6~~, 7, 8, 9.

$5\times7\times8\times9=2520.$

2 5 2 0 0 0 0 0 0

21. The digits 1,2,3,4, and 5 are used to make a five digit number PQRST. The three digit number PQR is divisible by 4, the three digit number QRS is divisibile by 5, and the three digit number RST is divisible by 3. What is P?

$R+S+T=3k$ ($k=1,2,3\cdots$)

$R+T+5=9, 12.$

QR = 12, 32, 24

① $R=2$, $T=2,5$ X.

② $R=4$, $T=3$. → 1 2 4 5 3.

22. When the three-digit number 5a2 is added to the number 247, the resulting number is a three-digit number 8b9. Find a + b if 8b9 is divisible by 9.

$5a2+247=8b9.$

$10a+749=10b+809.$

$10a-10b=60.$

$a-b=6.$

$8+b+9=18.$

$b=1.$

$\therefore a=7.$

$\therefore a+b=8.$

23. Let N = 2 4 36 510 7 9 .

a) How many factors does N have? $(6+1)\times(1+1)(1+1)=28.$

b) How many odd factors does N have? 28.

c) How many even factors does N have? 0.

d) How many perfect square factors does N have? $(1, 7^2, 7^4, 7^6)$. $\boxed{4}$

e) How many odd perfect square factors does N have? 4.

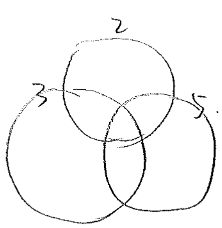
f) How many even perfect square factors does N have? 0.

g) How many factors of N are multiples of 7? $28-4=24.$

h) How many factors of N are not multiples of 7? (1, 19, 109, 19×109). 4

$243651079=7^6\cdot 19\cdot 109$

24. CHallenge: Call a number prime-looking if it is composite but not divisible by 2, 3, or 5. The three smallest prime-looking numbers are 49, 77, and 91. There are 168 prime numbers less than 1000. How many prime-looking numbers are there less than 1000? AMC12



$1000\div 2=500.$

$1000\div 3\approx 333$

$1000\div 5=200$

$1000\div(2\times 3)=166.$

$1000\div(3\times 5)=66.$

$1000\div(2\times 5)=100$

$1000\div(2\times 3\times 5)=33.$

$500+333+200-166-66-100+33=734.$

$1000-734-168+3$ (2, 3, 5)

$=1000-899$

$=\boxed{101}$